Lepton Masses and Mixings in Next-to-minimal Supersymmetric SO(10) GUT

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Abstract

A simple extension of the minimal renormalizable supersymmetric SO(10) grand unified theory by adding a 120-dimensional Higgs representation is examined. This brings new antisymmetric contributions to the relevant quark and lepton mass sum rules and leads to a better fit of the measured values of lepton masses and mixings together with a natural completion of the renormalizable Higgs sector within the SUSY SO(10) framework.

1 Introduction

The class of supersymmetric grand unified theories (GUT) based on the $SO(10)$ gauge group seems to be one of the most promising frameworks to describe the physics beyond the Standard model, including massive neutrinos. Though the scale at which the GUT symmetry should be realized is very large ($\sim 10^{16}$ GeV) there could be observable consequences of such scenarios at the laboratory energies, be it the tiny neutrino masses, measurable proton decay rate, anomalous electric dipole moments and other phenomena. The high scale symmetry propagates into the low-energy observables by means of effective relations among quantities which are in general uncorrelated within the SM framework. This makes such scenarios quite predictive and thus very attractive from the point of view of the low-energy phenomenology. In this talk we give a short overview of the minimal renormalizable SUSY SO(10)[1, 2, 4], in particular the effective mass sum rules for quark and lepton mass matrices arising from the GUT-scale physics and their consequences on the leptonic sector, namely the predictions for the neutrino masses and the PMNS lepton mixing matrix. Then we define a simple renormalizable extension of the minimal model by including one additional (almost decoupled) 120-dimensional Higgs multiplet. We argue that even a tiny admixture of its bidoublet components within the light MSSM Higgs doublets can lead to substantial effects in the predicted values of the neutrino masses and PMNS mixing angles.

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2 Minimal renormalizable SUSY SO(10) model

In the past few years one may notice a 'renaissance' [3, 4, 5, 6, 7, 8, 9] of the renormalizable SUSY SO(10) model. It was shown that this framework can accommodate good R-parity conserving $SO(10) \to SM$ breaking patterns being more constrained than any realistic GUT model based on the $SU(5)$ gauge group[4]. Moreover, there is an intriguing relationship [10] between the (approximate) maximality of the atmospheric mixing in the leptonic sector and the b-τ unification provided the neutrino mass matrix is dominated by the type-II seesaw contribution.

Structure of the minimal renormalizable SUSY SO(10)

One of the most appealing features of any $SO(10)$ GUT model is the fact that the SM fermions of each generation (including the right-handed neutrinos) reside in one irreducible 16-dimensional representation of the gauge group, the spinorial 16_F. Concerning the Yukawa part of the superpotential, the matter bilinear $16_F \times 16_F$ can couple at the renormalizable level only to three types of Higgs multiplets, namely the 10-dimensional vector multiplet 10_H, the 126-dimensional antiselfdual 5-index antisymmetric tensor $\overline{126}_H$ and the 120-dimensional three-index antisymmetric tensor 120_H. It was shown[4, 7] that in order to obtain a realistic leptonic spectrum it is sufficient to consider only the 10_H and $\overline{126}_H$ Higgs multiplets. (In addition the 210-dimensional four-index antisymmetric tensor 210_H is needed to break properly the $SO(10)$ group down to the MSSM and mix the 10_H and $\overline{126}_H$ to generate the left-handed triplet VEV entering the seesaw formula together with the proper mixings among the components entering the two light Higgs doublets of the MSSM).

Sum rules for quark and lepton mass matrices in MRM

Inspecting the $SU(3)_c \times SU(2)_L \times U(1)_Y$ structure of these multiplets one sees that the quark and lepton mass matrices obey

$$\begin{array}{rclr} M_u & = & Y_{10}v_u^{10} + Y_{126}v_u^{126} & M_d = Y_{10}v_d^{10} + Y_{126}v_d^{126} \\ M_l & = & Y_{10}v_d^{10} - 3Y_{126}v_d^{126} & M_\nu^D = Y_{10}v_u^{10} - 3Y_{126}v_u^{126} \\ M_\nu^R & \propto & Y_{126}\langle(1,1,0)_{\overline{126}}\rangle & M_\nu^L \propto Y_{126}\langle(1,3,+2)_{\overline{126}}\rangle \end{array} \quad (1)$$

provided Y_{10} and Y_{126} are the (symmetric) Yukawa matrices parametrizing the coupling of the matter bilinear to the relevant Higgs multiplets and $v_{u,d}^{10,126}$ are the VEVs of bidoublets contained in 10_H and $\overline{126}_H$.

Lepton mixing in MRM with dominant type-II seesaw

Assuming that the type-II contribution dominates the neutrino mass-formula one can obtain the following sum-rules for the charged lepton and neutrino mass matrices[7]

(in the basis where M_d is diagonal $\equiv D_d$; the tilde denotes the mass matrix normalized to its largest eigenvalue)

$$k\tilde{M}_l = V_{CKM}^T \tilde{D}_u V_{CKM} + r\tilde{D}_d \qquad M_\nu \propto \tilde{M}_l - \frac{m_b}{m_\tau}\tilde{D}_d \tag{2}$$

Here k and r are functions of the v-parameters in (1). Looking at (2) one can appreciate the predictivity of the model: *i)* It is very nontrivial to get a good fit of the charged lepton mass ratios at the LHS of (2) by varying only the quark masses and mixings within their experimental ranges and using the freedom in r and the remaining 6 complex phases (in $\tilde{D}_x$) on the RHS. *ii)* Whenever one finds a region in the parametric space where the charged lepton mass ratios fit well, the neutrino mass matrix is known up to just one phase. Thus the model is very predictive in the neutrino sector. Moreover, if m_b approaches m_τ (and the relative phase of the two terms is adjusted properly), the 33-entry of m_ν is comparable with the other entries in the 23 sector, what leads to the almost bimaximal structure of the U_{PMNS}[10]. Coming to the numerical analyses[7, 8] (with the CP-phases switched off for simplicity; their effects were shown to be subleading in most cases[7] usually worsening the fit of the charged lepton formula), the following predictions are obtained at the 1-σ level [7, 8]: $|U_{e3}| \geq 0.15$, $\sin^2 2\theta_{13} \geq 0.85$, $\sin^2 2\theta_{23} \leq 0.97$. The lower bound for $|U_{e3}|$ turns out to be very rigid and can be a 'smoking gun' of the model. Moreover, the solar mixing tends to be too large, while the atmospheric is never maximal.

3 Extending the minimal renormalizable model

Though the minimal model predictions are in a reasonably good agreement with the experimental data by extending the analysis to 2-σ level one may ask whether some extensions of MRM may perform better. However, often the price to be paid is the lack of predictivity and one should look for extensions that are constrained enough to remain as predictive as possible.

Adding a 120-dimensional Higgs representation

One of (few) such renormalizable generalizations of the MRM is the scenario with one additional quasidecoupled 120-dimensional Higgs representation (to which we refer as the next-to-minimal renormalizable model, NMRM)[8]. The key observation is that the 120_H multiplet can be naturally heavier than the GUT scale, because it does not participate at the GUT-symmetry breaking. Its scalar mass parameter M_{120} is not constrained by potential-flattness conditions and can be naturally as large as the cut-off, be it the Planck scale. This means that the weights of the

bidoublet components entering the weak-scale MSSM Higgs doublets may be naturally suppressed with respect to those coming from 10_H and $\overline{126}_H$. Therefore the relations (1) are only slightly modified thus preserving most of the good features of the MRM. Let us write down the new quark and lepton mass formulae :

$$\begin{aligned} M_u &= Y_{10}v_u^{10} + Y_{126}v_u^{126} + Y_{120}v_u^{120} \qquad M_d = Y_{10}v_d^{10} + Y_{126}v_d^{126} + Y_{120}v_d^{120} \\ M_l &= Y_{10}v_d^{10} - 3Y_{126}v_d^{126} + Y_{120}v_l^{120} \qquad M_\nu \propto Y_{126}\langle(1,3,+2)_{\overline{126}}\rangle \end{aligned} \tag{3}$$

The inequality $M_{120} \gg M_{GUT}$ translates into $v_x^{120} \ll v_{u,d}^{10,126}$. Equivalently, one can write (diagonalizing the quark mass matrices by means of biunitary transformations $M_x = V_x^R D_x V_x^{L^T}$, $x = u, d$ and denoting $W \equiv V_u^{R^T} V_d^R$, $V_{CKM} \equiv V_u^{L^T} V_d^L$ and $Y'_{120} \equiv V_d^{R^T} Y_{120} V_d^L$)

$$\begin{aligned} kV_d^{R^T}\tilde{M}_l V_d^L &= W^T \tilde{D}_u V_{CKM} + r\tilde{D}_d + Y'_{120}(k\varepsilon_l - \varepsilon_u - r\varepsilon_d) \\ M_\nu &\propto \tilde{M}_l - \frac{m_b}{m_\tau}\tilde{M}_d + Y_{120}\left(\frac{m_b}{m_\tau}\varepsilon_d - \varepsilon_l\right) \end{aligned} \tag{4}$$

with $\varepsilon_{u,d,l} \equiv v_{u,d,l}/m_{t,b,\tau}$. Since Y_{120} is antisymmetric, the $M_{u,d,l}$ are no longer symmetric and the unknown right-handed quark mixing matrix W appears at the RHS of (4). However, since this setup is a perturbation of the MRM one can expand the W matrix around V_{CKM} by means of the small parameters in the game (neglecting the CP phases): $W = V_{CKM} + 2\left(-\varepsilon_u Z_u V_{CKM} + \varepsilon_d V_{CKM} Z_d\right) + O(\varepsilon_x^2)$ where the Z_x matrices are given by $(Z_x)_{ij} = (Y'_x)_{ij}/\left[(\tilde{D}_x)_{ii} + (\tilde{D}_x)_{jj}\right]$ and $Y'_u \equiv V_{CKM} Y'_{120} V_{CKM}^T$, $Y'_d \equiv Y'_{120}$. Therefore, for small ε_x the predictions of this model are expected to be close to those of MRM. It can be shown that the tiny antisymmetric corrections change the PMNS angles linearly in $\varepsilon's$ while the masses are affected at the second order, what makes the perturbative method self-consistent and the fit of the charged lepton mass matrix stable enough to preserve the good features of the MRM. On the other hand, the 1-2 entries of the Z_x matrices can be strongly enhanced by the small '$\tilde{D}_{11,22}$' terms in the denominator.

Lepton mixing in NMRM with dominant type-II seesaw

The numerics shows[8] that even for $\varepsilon \sim 10^{-3}$ the non-decoupling effects of the 120_H contributions to the $PMNS$ angles can reach several tens of percent. For instance the MRM lower bound for $|U_{e3}|$ can be relaxed to about $|U_{e3}| > 0.1$, while the atmospheric angle can be maximal and the solar bound is changed to about $\sin^2 2\theta_{13} > 0.75$, all this at 1-$\sigma$ level even with the CP-phases switched off.

4 Conclusions

We have argued that the lepton sector predictions of the minimal renormalizable SUSY SO(10) model are very sensitive to the magnitude of the antisymmetric Yukawa structure be it an additional small renormalizable coupling of 120_H Higgs multiplet to the matter bilinear (or an effective vertex generated by dynamics beyond the GUT-scale). Therefore such terms should be taken into serious consideration when discussing the phenomenological implications of such class of grand unified theories.

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